A Concise Introduction to Random Number Generation

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Overview of This Talk

How to assess RNGs?

- criteria
- a checklist

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- a checklist

A note on statistical testing

- strategies
- Maurer's Universal Test and related tests

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Interesting RNGs

- AES
- HAVEG(E)

RNGs: The Goal

What we want . . .

A device (hardware or software) whose output is random.

RNGs: The Goal

What we want ...
A device (hardware or software) whose output is random.

More precisely ...
Want to generate bits (or numbers) that appear like being sampled from a uniform distribution on $\{0,1\}$ (or $[0,1[$), independently of each other.

RNGs: The Reality

What we get. . .

Finite output streams that pass many tests of randomness.

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Deterministic algorithm whose output mimics finite random sequences.

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Finite output streams that pass many tests of randomness.

Pseudorandom number generator (PRNG)

Deterministic algorithm whose output mimics finite random sequences.

Question

What are random sequences?

Randomness

Quote

> *"A finite sequence is random if there is no short sequence that describes it fully, in some unambigous mathematical notation."*

. . . A. Kolmogoroff

Quote

> *"A string is random if it cannot be algorithmically compressed."*

. . . C. Calude

Remark

The basic idea of Kolmogoroff complexity:

Randomness = Incompressibility

RNGs: Practice

Quote

> *"Monte Carlo results are misleading when correlations hidden in the random numbers and in the simulated system interfere constructively."*

. . . A. Compagner, Phys. Rev. E **52**(1995)

Quote

> *"Ironically, pseudorandom numbers often appear to be more random than random numbers obtained from physical sources."*

. . . A. Rukhin et al., NIST Special Publ. 800-22

RNGs: An Illustration

With RNGs, there are no guarantees.

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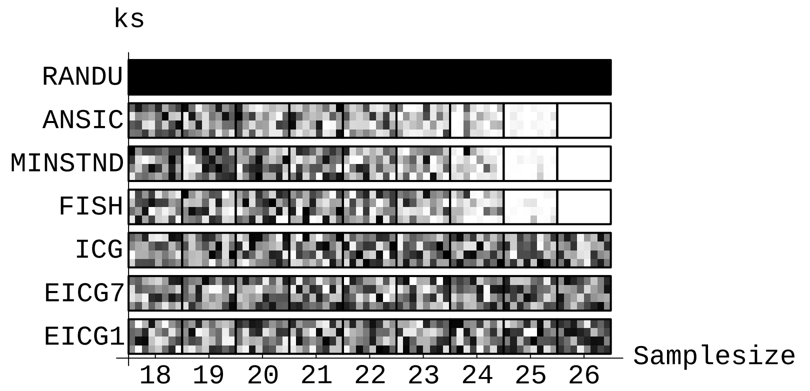

True Value:	Irregular Pattern in Every Box
RNGs:	LCGs and (E)ICGs
Sample Size:	$2^{18}..2^{26}$

Phenomena

Setup

- RNG: LCG(2^{31}, 65539, 0, 1), i.e. RANDU

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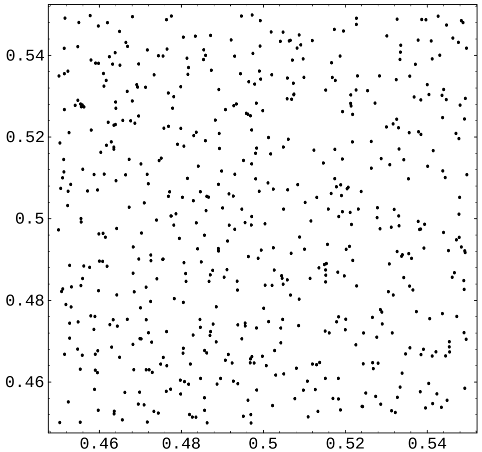
Phenomena

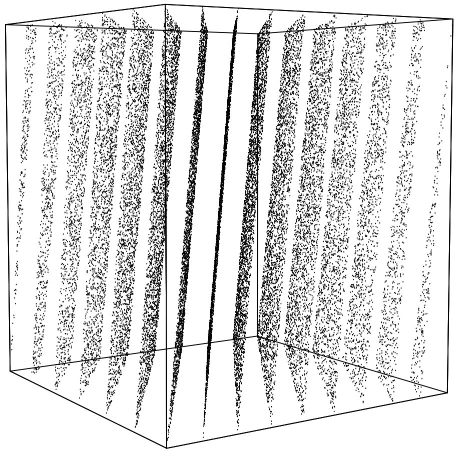
Setup

- RNG: LCG(2^{31}, 65539, 0, 1), i.e. RANDU
- Dimension: $d = 2, 3$
- Sample size: $N = 2^{16}$
- Plot nonoverlapping pairs (x_{2n}, x_{2n+1}) and triples $(x_{3n}, x_{3n+1}, x_{3n+2})$, $0 \leq n < N$.

Phenomena: Increasing the Dimension

We increase the dimension from $d = 2$ to $d = 3$:





Question

How to prevent such unpleasant surprises?

Phenomena: Increasing the Dimension

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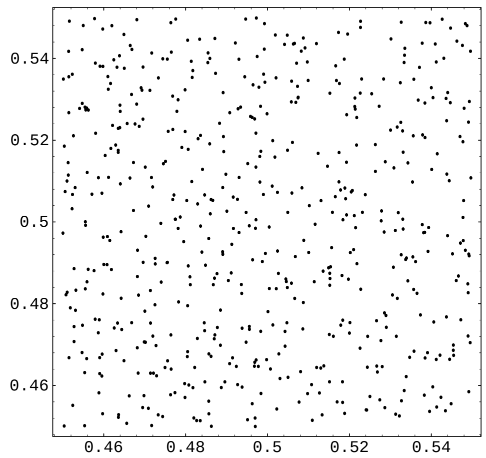

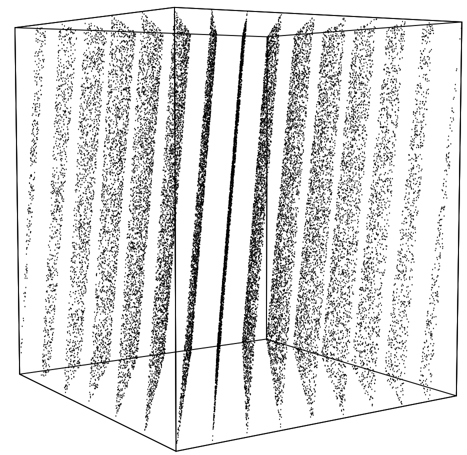

Question

How to prevent such unpleasant surprises?

Answer

Theoretical correlation analysis and/or statistical testing.

Linear Congruential Generator (LCG)

Parameters

- m . . . modulus

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Defining congruence

$$y_{n+1} \equiv a \cdot y_n + b \pmod{m}, \quad n \geq 0$$

...LCG(m, a, b, y_0)

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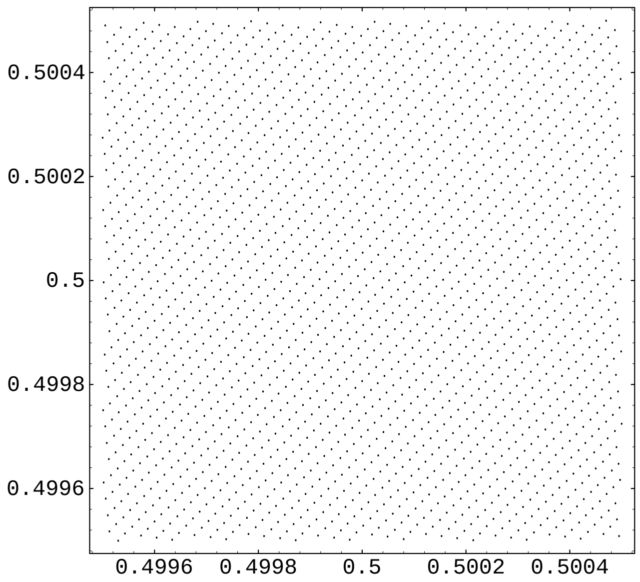
. . . LCG(m, a, b, y_0)

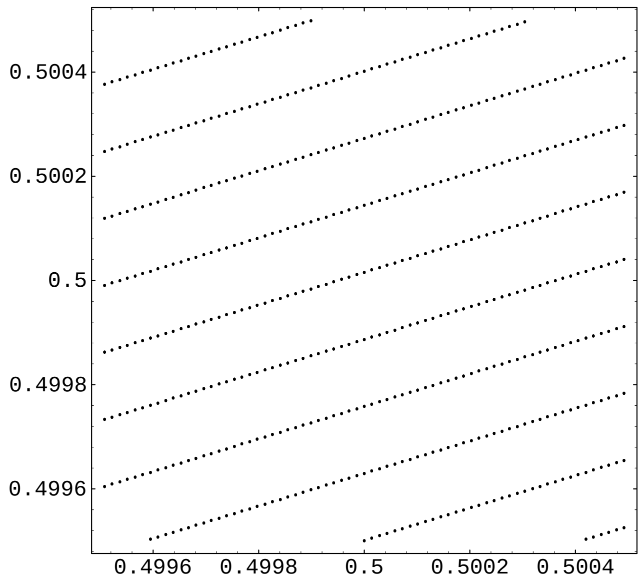
Output stream

$$x_n := \frac{y_n}{m} \in [0, 1[, \quad n = 0, 1, \ldots$$

LCGs: Two Examples



LCG($2^{31} - 1$,630360016,0,1)



LCG(2^{32}, 69069, 0,1)

Inversive Congruential Generator (ICG)

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- m ... modulus (usually a big prime)

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$$y_{n+1} \equiv a \cdot \overline{y_n} + b \pmod{m}, \quad n \geq 0$$

($\bar{c} = c^{-1}$ for $c \neq 0$, $\quad \bar{c} = 0$ if $c = 0$.)

... ICG(m, a, b, y_0)

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Output stream

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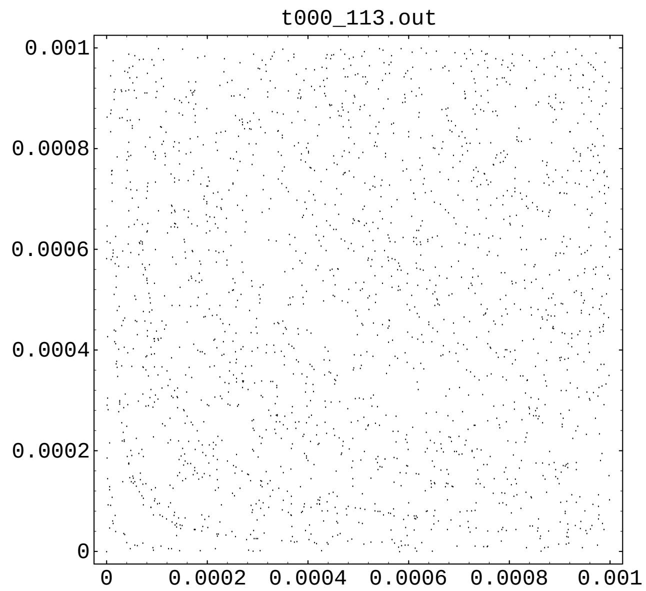
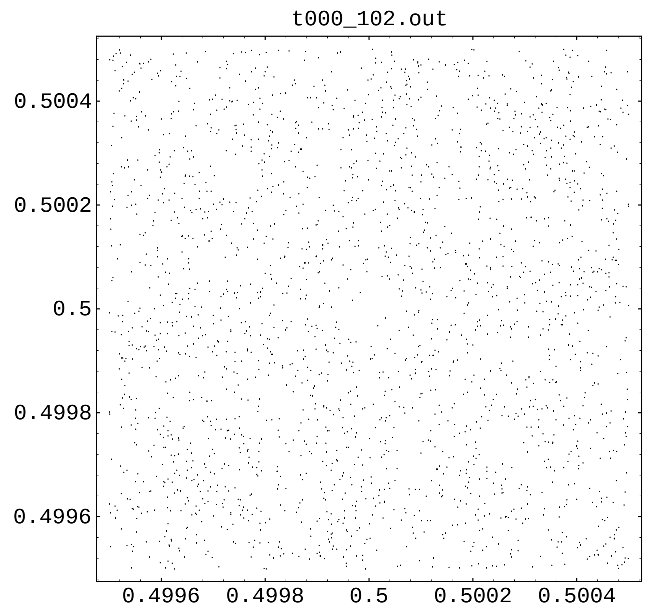
ICG: Point Structure

ICG($2^{31} - 1$, 1288490188, 1, 1)



lower left corner



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Pseudorandom Number Generators (PRNGs)

PRNG: A tuple $G = (\mathcal{S}, \mathcal{I}, T, \mathcal{O}, g, s_0)$, where

- $\mathcal{S}$ is the finite state space,

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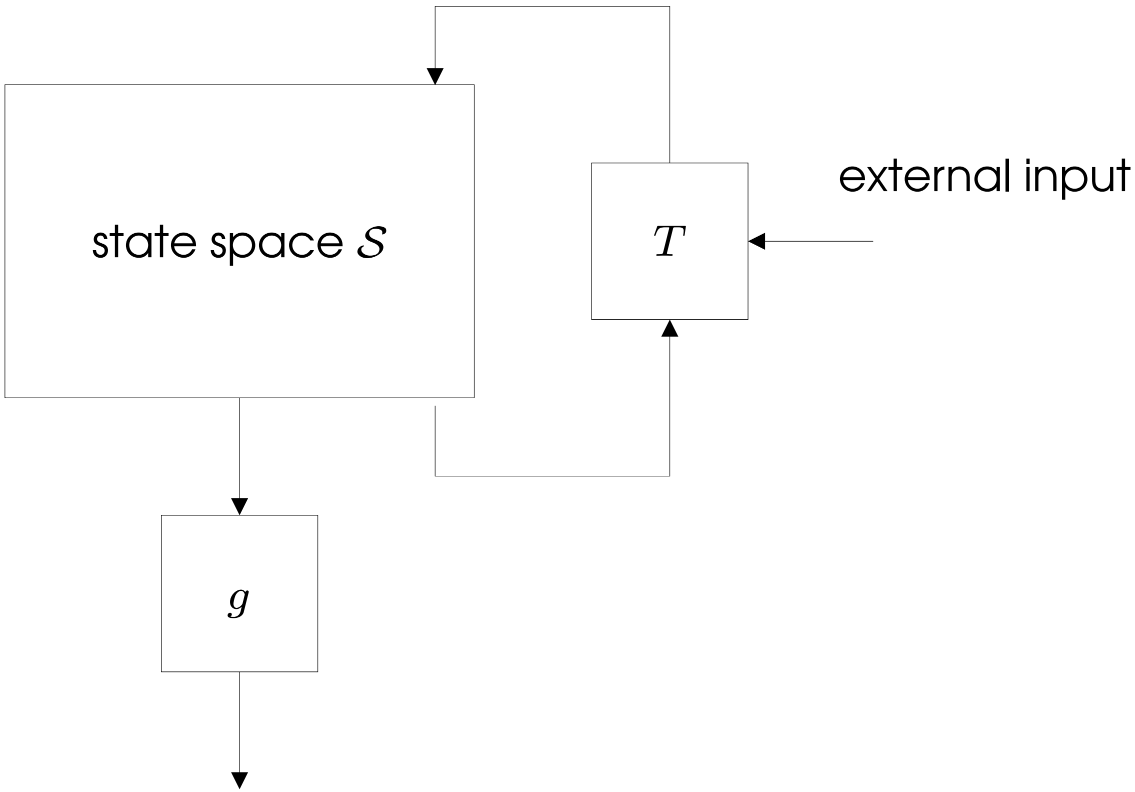
The next state s_{n+1} is generated by

$$s_{n+1} = T(i_n, s_n), \qquad n \geq 0,$$

the output stream $(o_n)_{n\geq 0}$ is computed by

$$o_n = g(s_n), \qquad n \geq 0.$$

PRNGs: The Structure



The Structure of a PRNG

Classification of RNGs

Types of RNGs

Type of Application	
Simulation (Monte Carlo)	Cryptography (stream ciphers)

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Classes of PRNGs

PRNGs: Type of Algorithm	
linear	nonlinear

Which RNG?

RNG vs. Application

RNG \ Application	Simulation	Cryptography
Hardware	not recommended	task dependent
Software	recommended	task dependent

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PRNG vs. Application

PRNG \ Application	Simulation	Cryptography
Linear	recommended (if chosen properly)	not recommended (insecure)
Nonlinear	task dependent (too small, too slow)	recommended (if chosen properly)

Checklist: Theoretical Support

A) Theoretical Support		
Period Length	Conditions	
	Algorithms for parameters	
Structural Properties	Intrinsic structures	
	Equidistribution properties	
	Predictability	
Correlation Analysis	For particular parameters	
	For particular initializations	
	For parts of the period	
	For subsequences	
	For combinations of RNGs	

Checklist: Statistical Testing

B) Statistical Testing	
Variable sample size Two- or higher level tests	
Performance with test batteries	
Serial test family	
Return times	
Other test quantities	
Transformation methods: sensitivity	

Checklist: Practical Aspects

C) Practical Aspects	
Tables of parameters available?	
Portable implementations available?	
Parallelization techniques applicable?	
Large samples available?	
Efficiency?	
Cryptography: security aspects?	

RNGs: Simulation vs. Cryptography

Simulation	**Cryptography**
Theoretical Analysis	
Period Length	
Known (in most cases)	Unknown (in most cases)
Structural Properties	
Intrinsic structures welcome	Intrinsic structures are to be avoided

RNGs: Simulation vs. Cryptography

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Structural Properties	
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Statistical Testing	
Extensive results	Lack of published test results
Batteries of tests	Under development (NIST)

RNGs: Simulation vs. Cryptography

Simulation	**Cryptography**
Practical Aspects	
RNGs trimmed for efficiency	RNGs in many flavors

RNGs: Simulation vs. Cryptography

Simulation	**Cryptography**
Practical Aspects	
RNGs trimmed for efficiency	RNGs in many flavors
Design Aspects	
Prefer linear algorithms (efficiency!)	Require nonlinear algorithms (security!)

RNGs: Simulation vs. Cryptography

Simulation	**Cryptography**
RNG Testing	
fair adversary: RNG treated as a black box	freestyle: all attacks allowed
test tries to find structures in the output stream	same goal
not interested in predictability	try to find the secret key

NIST Test Suite (NTS)

Comments

- Question I:
  What are the redundancies in this test suite?
  For example, NST contains various entropy estimators (Maurer's Universal Test, Approximate Entropy of Pincus and Singer, Serial Test). What is the relation between them?

NIST Test Suite (NTS)

Comments

- Question I:
  What are the redundancies in this test suite?
  For example, NST contains various entropy estimators (Maurer's Universal Test, Approximate Entropy of Pincus and Singer, Serial Test). What is the relation between them?
- Question II
  Which NIST tests detect which kind of defect?
  The NTS has not been analyzed with respect to a defective RNG. Which tests will detect a given defect (and which tests will not)?

Testing Statistical Tests

Question

How universal is Maurer's Universal Test?

Testing Statistical Tests

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How universal is Maurer's Universal Test?

Approach

- Construct bitstream $x_0, x_1, \ldots$
  induce correlations at distance κ:

$$\boxed{x_0},\ x_1,\ x_2,\ \ldots,\ x_{\kappa-1},\ \boxed{x_\kappa},\ x_{\kappa+1},\ \ldots$$

- Does the statistical test at hand detect this error?

Testing Statistical Tests

Question

How universal is Maurer's Universal Test?

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Results

See our "Defective Source Analysis"

Defective Source Analysis

Correlations

Choose order κ, $\kappa \geq 1$

Choose random bits

$$x_0, x_1, \ldots, x_{\kappa-1}$$

Defective Source Analysis

Correlations

Choose order κ, $\kappa \geq 1$

Choose random bits

$$x_0, x_1, \ldots, x_{\kappa-1}$$

Choose bias λ

$$x_i = \begin{cases} x_{i-\kappa} & \text{with probability } \lambda \\ 1 - x_{i-\kappa} & \text{with probability } 1 - \lambda \end{cases}$$

Defective Source Analysis

Correlations

Choose order κ, $\kappa \geq 1$

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Choose source probability distribution

$\lambda = 0.5$ . . . i.i.d. uniform

$\lambda \neq 0.5$ . . . i.d. uniform

Defective Source Analysis

- Test input $x_0, x_1, \ldots, x_{m-1}$ (m bits)

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- Overlapping and non-overlapping d-tuples

$$\tilde{x}_i^d = (x_i, x_{i+1}, \ldots, x_{i+d-1})$$

$$\bar{x}_i^d = (x_{i \cdot d}, x_{i \cdot d+1}, \ldots, x_{i \cdot d+d-1})$$

Defective Source Analysis

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- Frequency count

$$\mathbf{a} = (a_1, \ldots, a_d) \in \{0,1\}^d$$

$$\tilde{\pi}_{\mathbf{a}}^d = \frac{1}{n} \#\{0 \leq i < n : \tilde{x}_i = \mathbf{a}\}$$

$$(\bar{\pi}_{\mathbf{a}}^d = \frac{1}{n} \#\{0 \leq i < n : \bar{x}_i = \mathbf{a}\})$$

Defective Source Analysis

Approximate Entropy

$$\hat{H}_f^d = -\sum_{\mathbf{a}\in\mathcal{A}^d} \tilde{\pi}_{\mathbf{a}}^d \log \tilde{\pi}_{\mathbf{a}}^d + \sum_{\mathbf{a}\in\mathcal{A}^{d-1}} \tilde{\pi}_{\mathbf{a}}^{d-1} \log \tilde{\pi}_{\mathbf{a}}^{d-1},$$

$$\hat{I}^d = 2n(1-\hat{H}_f^d) \xrightarrow{\mathrm{D}} \chi^2_{2^d-2^{d-1}}$$

. . . (Pincus and Singer, 1998)

Defective Source Analysis

Approximate Entropy

$$\hat{H}^d_f = -\sum_{\mathbf{a}\in\mathcal{A}^d} \tilde{\pi}^d_{\mathbf{a}} \log \tilde{\pi}^d_{\mathbf{a}} + \sum_{\mathbf{a}\in\mathcal{A}^{d-1}} \tilde{\pi}^{d-1}_{\mathbf{a}} \log \tilde{\pi}^{d-1}_{\mathbf{a}},$$

$$\hat{I}^d = 2n(1 - \hat{H}^d_f) \xrightarrow{\mathrm{D}} \chi^2_{2^d - 2^{d-1}}$$

. . . (Pincus and Singer, 1998)

Universal Test

$$\hat{H}^d_r = \frac{1}{d \cdot n} \sum_{i=Q}^{Q+n-1} \log T(i)$$

$$\hat{N}^d = \frac{\hat{H}^d_r - E[\cdot]}{\sqrt{V[\cdot]}} \xrightarrow{\mathrm{D}} N[0,1]$$

. . . (Maurer, 1992)

($T(i)$: return time for $\bar{x}^d_i$)

Defective Source Analysis

Overlapping Serial Test

$$\hat{\chi}^d = n \sum_{\mathbf{a} \in \mathcal{A}^d} \frac{(\tilde{\pi}^d_{\mathbf{a}} - (1/2)^d)^2}{(1/2)^d} - n \sum_{\mathbf{a} \in \mathcal{A}^{d-1}} \frac{(\tilde{\pi}^{d-1}_{\mathbf{a}} - (1/2)^{d-1})^2}{(1/2)^{d-1}}$$

. . . (I.J. Good, 1953)

Defective Source Analysis

Overlapping Serial Test

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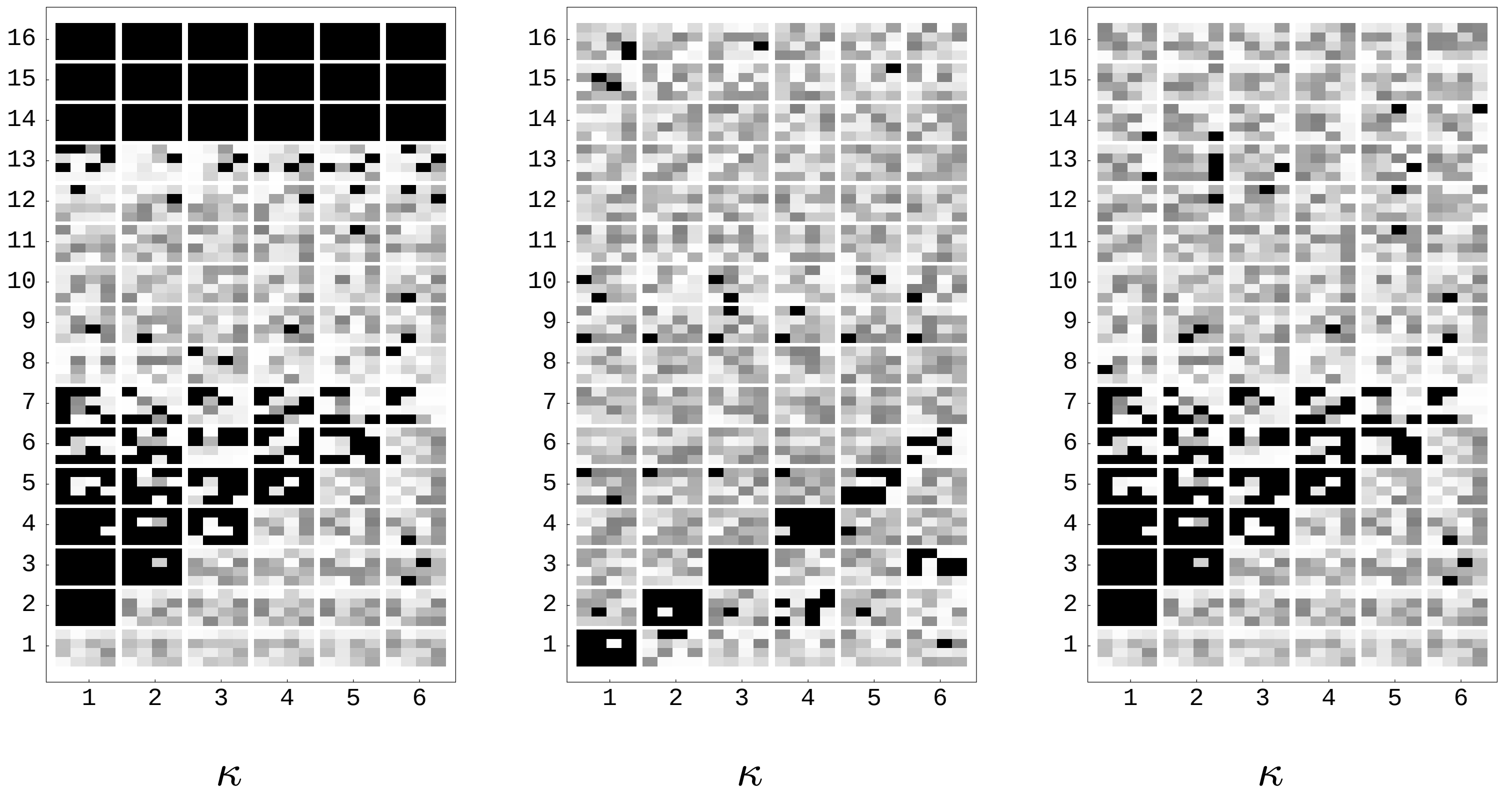
Test Parameters

Sample size	$n = 2^{16},\ 2^{18}$ bits
No. of repetitions	16 indept. samples
Dimension	$d = 1..16$
Order	$\kappa = 1..6$
Bias λ	$\lambda = 0.49$
Entropy of source	$H \approx 0.999711$

Defective Source Analysis

Results for $n = 2^{16}$ bits

Black dots denote p-values smaller than 0.01.



$\hat{I}^d$, ApEn $\hat{N}^d$, Universal Test $\hat{\chi}^d$, Overl. Serial T.

Defective Source Analysis

Results for $n = 2^{18}$ bits

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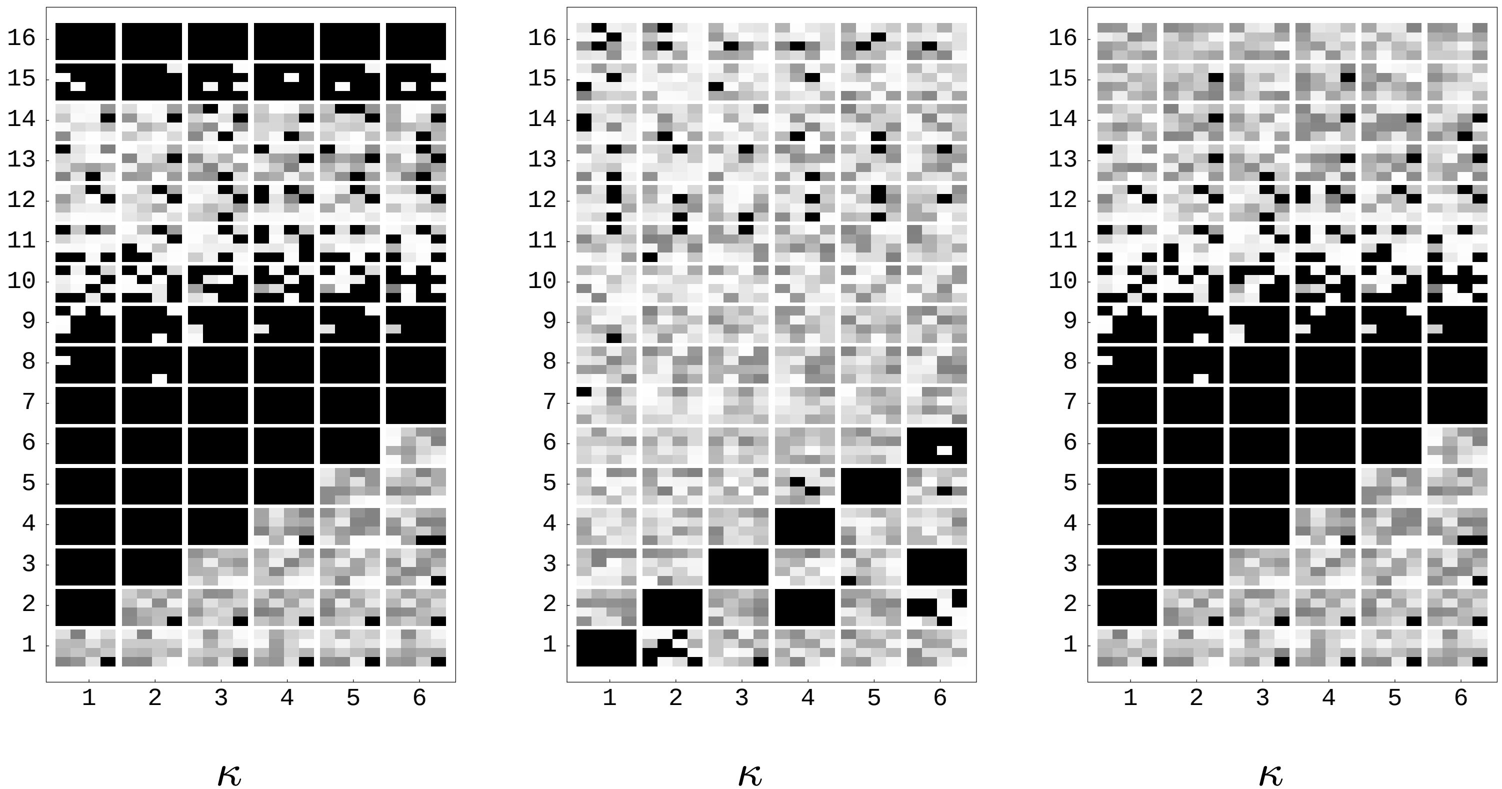

$\hat{I}^d$, ApEn | $\hat{N}^d$, Universal Test | $\hat{\chi}^d$, Overl. Serial T.

AES: Modes of Operation

Output Feedback Mode MODE (OFB)

choose	k	. . . key
choose	z_0	. . . initial value
compute	$\left(e_k^{(i)}(z_0)\right)_{i\geq 0}$	. . . output stream

$$\underbrace{e_k^{(i)} = e_k \circ \ldots \circ e_k}_{i \text{ times}}$$

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COUNTER MODE

choose	k	...key
compute	$x_0, x_1, \ldots$ (counter)	...plaintext
compute	$(e_k(x_i))_{i\geq 0}$	...output stream

AES: Setups

Setup 1 – PRNG

$k,\ z_0$ ... various cases (all-zero, random, ...)

$\left(e_k^{(i)}(z_0)\right)_{i\geq 0}$ ... output stream

AES: Setups

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$\left(e_k^{(i)}(z_0)\right)_{i\geq 0}$	...output stream

Setup 2 – DIFF

k	...various cases (all-zero, random, ...)
$(p_i)_{i\geq 0}$	...highly patterned plaintext blocks
$(e_k(p_i))_{i\geq 0}$	...output stream

AES: Setups

Setup 1 – PRNG

$k,\ z_0$	. . . various cases (all-zero, random, . . .)
$\left(e_k^{(i)}(z_0)\right)_{i\geq 0}$	. . . output stream

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k	. . . various cases (all-zero, random, . . .)
$(p_i)_{i\geq 0}$	. . . highly patterned plaintext blocks
$(e_k(p_i))_{i\geq 0}$	. . . output stream

Setup 3 – PCOUNT

k	. . . various cases (all-zero, random, . . .)
$(p_i)_{i\geq 0}$	. . . increasing counter
$(e_k(p_i))_{i\geq 0}$	. . . output stream

AES: Setups

Setup 1 – PRNG

$k, \; z_0$ ...various cases (all-zero, random, ...)

$\left(e_k^{(i)}(z_0)\right)_{i \geq 0}$ ...output stream

Setup 2 – DIFF

k ...various cases (all-zero, random, ...)

$(p_i)_{i \geq 0}$ ...highly patterned plaintext blocks

$(e_k(p_i))_{i \geq 0}$ ...output stream

Setup 3 – PCOUNT

k ...various cases (all-zero, random, ...)

$(p_i)_{i \geq 0}$ ...increasing counter

$(e_k(p_i))_{i \geq 0}$ ...output stream

Setup 4 – KCOUNT

p_0 ...plaintext block

$(k_i)_{i \geq 0}$ ...incrementing counter

$(e_{k_i}(p_0))_{i \geq 0}$ ...output stream

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Test Parameters

Sample size	$n = 2^{18}, 2^{19}, \ldots, 2^{28}$ bits
No. of repetitions	16 indept. samples
Dimension	$d = 1, 2, 4, 8, 16$

AES: Test II

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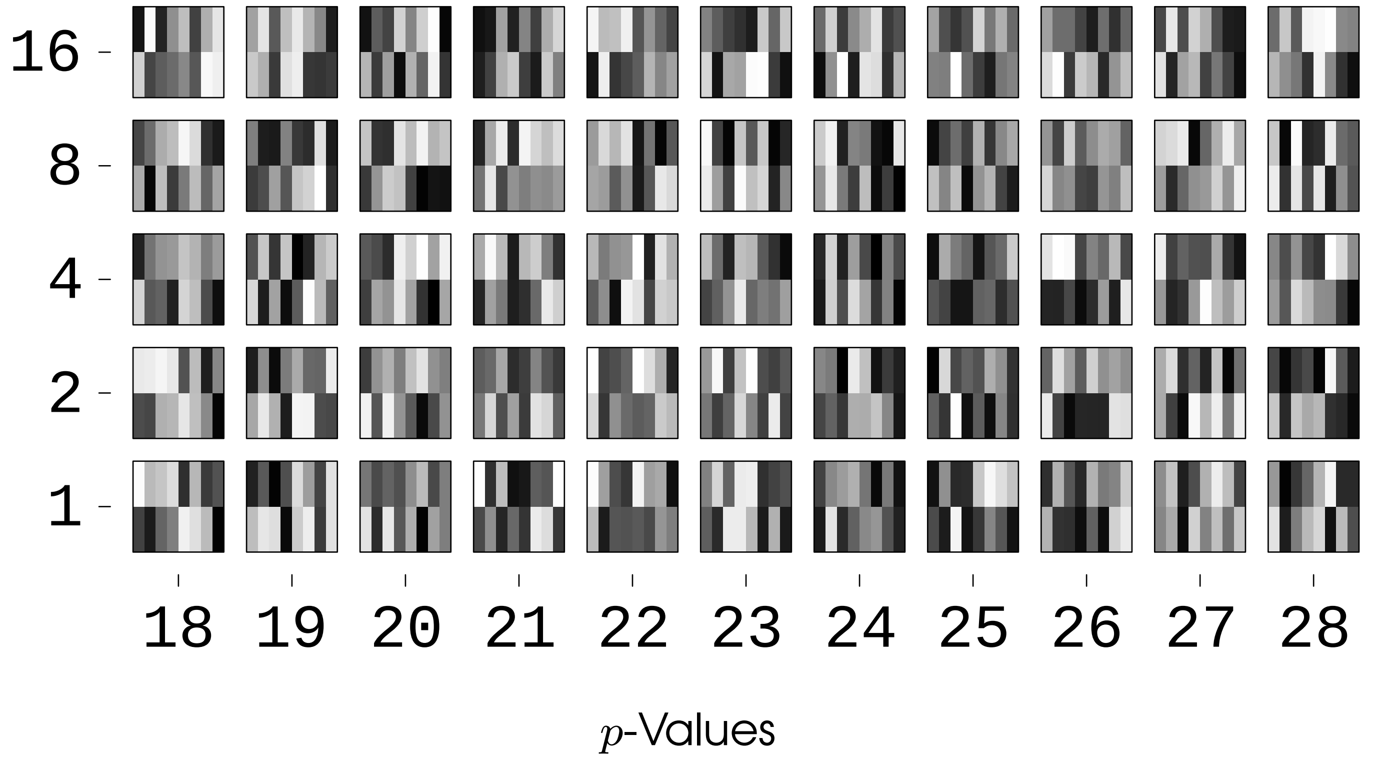
Test Parameters

Sample size	$n = 2^{22}, 2^{23}, \ldots, 2^{28}$ bits
No. of repetitions	16 indept. samples
Dimension	$d = 32, 64, 128, 256$

AES: Results of Test I



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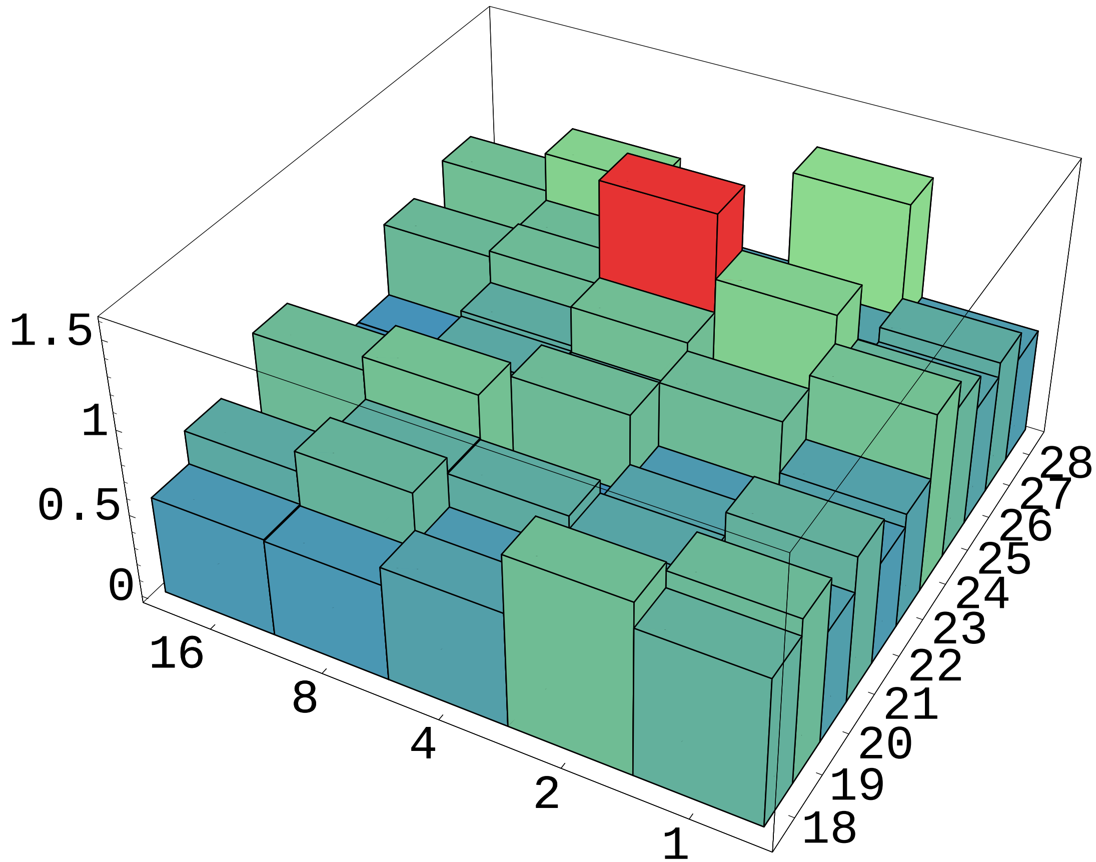

KS-Test Values

HAVEG and HAVEGE

HArdware Volatile Entropy Gathering and Expansion

Sendrier and Seznec (INRIA, 2002)

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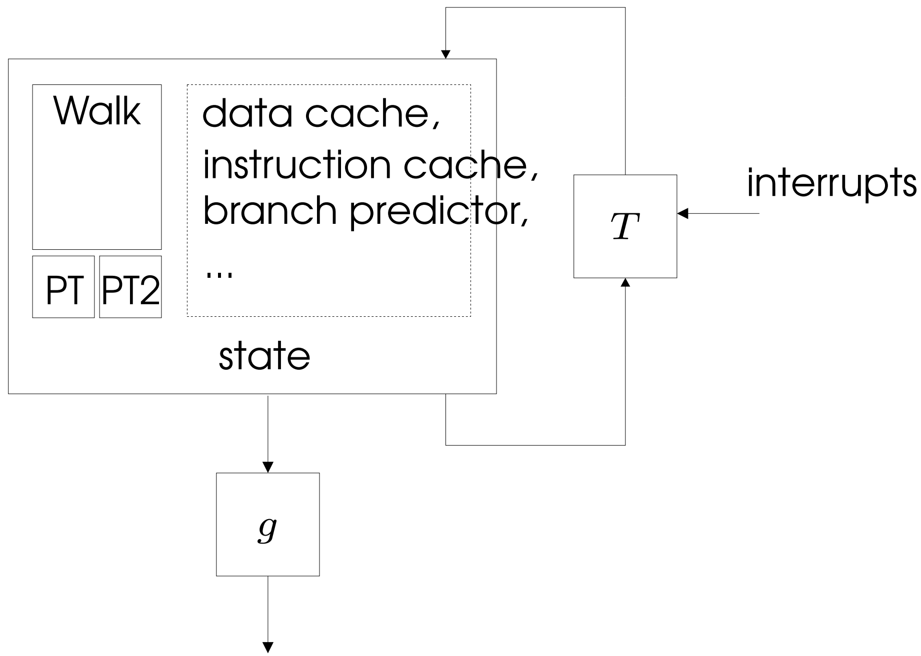
- HAVEGE is active, acts on the processor

The idea behind

Each attempt to read the inner state of the processor alters it at the same time.

Complete state of HAVEGE cannot be observed without freezing the clock of the processor.

Structure of HAVEGE



The General Structure of HAVEGE

RNGs: State of the Art

Present Situation

Like in the race between cryptographers and cryptanalysts, presently the designers of RNGs are winning against the designers of statistical tests.

The intrinsic structures of modern RNGs, in particular of good cryptographic RNGs, are too complicated to be detected by current statistical tests.

Future developments

New ideas for testing are needed. This will take some time.